20000726117

AD A053033

AD NO.
DDC FILE COPY

Reproduced From
Best Available Copy

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 HDL-TR-1827	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 7
4. TITLE (and Subtitle) 6 Fuzing Logic for an Earth-Penetrator Weapon against an Underground Target.		5. TYPE OF REPORT & PERIOD COVERED Technical Report.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Barbara A. Quinnell		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harry Diamond Laboratories 2800 Powder Mill Road Adelphi, MD 20783		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 16 DA: 1W662616AH77 Program Ele: 6.26.16.A
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Materiel Development and Readiness Command Alexandria, VA 22333		12. REPORT DATE 11 March 1978
		13. NUMBER OF PAGES 43
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) 12 43p.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

HDL Project: A776W1
DRCMS Code: 662.616.11.H7700

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Fuzing	Earth penetrator
Fuze logic	Weapon fuze
Computer fuze	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A model is developed for the fuzing logic required for an earth-penetrator weapon (EPW). Given missile guidance parameters and collateral damage constraints as initial conditions, further necessary parameters for defining a fuzing algorithm are derived from a geometric analysis of possible trajectories. Then the fuzing logic for such a weapon is determined for both linear and nonlinear underground trajectories. Assuming the missile remains

next page

1R

underground after impact, several fuzing logic criteria are given, depending on such factors as path form, vector miss distance, and impact angle.

White Section
Buff Section

A

DDC
APR 21 1978
D

CONTENTS

APPENDICES

TABLES

FIGURES

FIGURES (Cont'd)

1. INTRODUCTION

For guided missiles targeted against certain classes of targets, underground detonation of the warhead may be more effective than air burst in achieving a desired level of damage. Also, underground detonation can reduce the possibility of unacceptable nonmilitary casualties and collateral damage. The target is assumed to be finite in the sense that the size of the target is small in relation to the standard deviation of the distribution of miss distances possible. If the guidance system is such that no information on the vector miss distance between the weapon and the finite target is available or can be derived, no target-oriented warhead-detonation logic can be prescribed. Thus, for this case the warhead-detonation command must be derived from environmental information obtained near the target. For example, the fuzing logic for this application could be to command detonation at a fixed time interval after earth entry. With use of integrating accelerometers, the fuzing logic could command detonation after a prescribed travel distance or at a prescribed depth below the surface. None of these techniques define such (possibly more desirable) detonation points as the point of closest approach (PCA) between missile and target (for isotropic warheads) or within an acceptable isodamage contour (for nonisotropic warheads).

However, if missile guidance parameters with respect to the target are available for the pre-entry portion of the trajectory, along with appropriate sensors for measuring trajectory after earth entry, the problem of fuzing at an appropriate point defined with respect to the target becomes deterministic. It is the purpose of this study to examine the fuzing requirements analytically, and to derive the necessary relationships and logic for fuzing an earth penetrator weapon. The weapon is assumed to be deployed against an underground target or against a target where below-the-surface detonation could result in increased effectiveness or in a reduction in collateral damage, while still achieving a desired level of kill. For the purpose of this study we assume some form of terminal guidance that is target oriented. That is, an estimate of the vector miss distance between the missile and the designated target is available or can be derived from data which are also used for terminal guidance. It is also assumed that this information can be provided for the fuze processor at the time the measurements are made or within a few milliseconds.

Certain factors affecting the desired performance of the system are assumed either to be known before launch and to be stored in the fuze memory, or to be provided to the fuze by a communications link before earth entry. One such factor affecting the system performance may be a minimum desired burst depth below the surface, to satisfy some constraints on collateral damage, that is, damage to possibly nonmilitary facilities near the prescribed target. A second factor assumed to be known to the fuze is the location of the target with respect to the guidance aim point. For example, in the case of a target completely below the surface, the guidance miss vector, for the type of guidance system assumed here, would be measured to some point on the surface of the earth. The location of the desired target may be at the same earth coordinates (latitude and longitude) as the guidance aim point, and at a known depth, or it may be offset by a known distance and direction from the guidance aim point. This information would have to be made available to the fuze in the form necessary to permit use of the algorithms based on the intercept geometry logic described below. Another factor assumed to be known to the fuze beforehand is the information from which the desired burst-point requirements may be determined and used in the fuze logic. The fuze burst-point logic may also require data from the fuze sensor to function as desired.

This report will consider several cases, depending on the form of the underground trajectory, the type of target, and the form of the available initial data. First the initial

parameters as derived from guidance data are discussed, including the assumptions as to the form in which they are available and what further parameters can be derived from them. It is assumed in general that the missile remains underground after impact, and that there are two known depths: a minimum desired burst depth, m, and another depth, d, to the location of the defined target or to the coordinates of the center of some isovulnerable volume. The time parameter is defined to be zero on impact, and increasing thereafter. For the purpose of this analysis, the target is assumed to be at or below the aim point; any other case would require adding a coordinate translation to the derived relationships.

For all cases considered, a fuzing logic is defined, based on the initial conditions, underground trajectory, target position, and collateral damage criteria. Two different criteria for target destruction are assumed. The first is that detonation occurs at the PCA between missile and target. The second is that detonation occurs within some lethal contour, defined in terms of warhead directivity, and, if applicable and available, soil characteristics. The effect on the logic of imperfect data, imperfect measurements, and incomplete information for a deterministic solution is also studied.

In the analysis that follows, several simplified targets and underground trajectories are considered. The intercept geometry is first analyzed for a linear underground path, and then generalized to a path created by a generalized round acceleration. The target, as a first step, is considered to be either a point or a line. Each target case is then expanded. The point target is generalized to an isovulnerable spherical contour; the line target is generalized to an isovulnerable cylindrical contour.

2. INITIAL CONDITIONS

In general, the coordinate system is set up as follows. The origin is on the designated target for the point target case. The necessary modifications of this choice for linear targets and isovulnerable spherical and cylindrical contours are noted where relevant. The x_0 axis is the surface line from the aim point to the impact point. For this analysis, the surface is considered flat in a sufficiently large neighborhood of the aim point. The z axis is the surface normal that passes through the aim point and the target. Next the y_0 axis is chosen so that x_0-y_0-z forms a mutually perpendicular right-handed triple. In this fashion the x_0-y_0 plane defines the surface plane in a neighborhood of the aim point. Finally, the x_0 and y_0 axes, when translated downward along the z axis to the target origin, become the x and y axes. This construction is diagrammed in figure 1.

With respect to this system, the necessary parameters for this analysis can be defined more precisely. The aim point is the point $(0,0,d)$ and the point of impact is $(R,0,d)$ where R is the miss distance and d is the depth to the target. The vector miss distance has length R and is directed from the aim point to the impact point. The further parameters which will be needed for this analysis are the impact angle with respect to the surface α, the map coordinate system angles σ and γ, impact velocity V, deceleration A, and the surface angles θ' and θ.

2.1 Parameters θ' and θ

The value of θ' is required to determine whether the impact is early or late. The value of θ is needed in the computations of the optimal time for detonation. These two parameters are derived from the guidance data as follows.

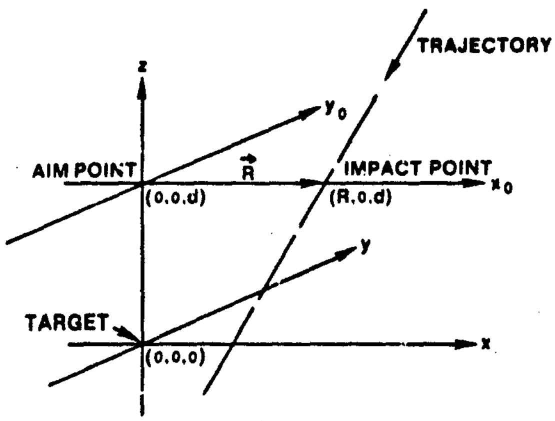

Figure 1. Intercept geometry.

The missile is assumed to have sensing mechanisms that can determine the angle, γ, of the miss distance vectors, $\vec{R}$, with respect to the map coordinate system of the aim point. Angle γ is defined as the angle measured from north to $\vec{R}$ counterclockwise, and is assumed to be always positive. Further, the sensor can determine the angle, σ, of the surface component, $\vec{S}$, of the trajectory with respect to the map coordinate system. Similarly, σ is the angle from north to $\vec{S}$ counterclockwise; again, σ is always positive. Angle θ' is defined to be the smaller angle between $\vec{R}$ and $\vec{S}$.

Figure 2 shows one possible relationship between σ, γ, and θ' (see app A for diagrams illustrating all such possible relations). Upon examination of these diagrams, the parameter θ' is calculated as follows.

$$\text{If } 0 \le |\sigma - \gamma| \le \pi, \quad \text{then} \quad \theta' = |\sigma - \gamma|, \text{ or} \tag{1}$$

$$\text{if } \pi < |\sigma - \gamma| \le 2\pi, \quad \text{then} \quad \theta' = 2\pi - |\sigma - \gamma|. \tag{2}$$

In all cases $0 \le \theta' \le \pi$.

Next, the angle θ is defined from θ' by

$$\theta = \theta' \quad \text{if} \quad 0 \le \theta' \le \pi/2, \text{ or} \tag{3}$$

$$\theta = \pi - \theta' \quad \text{if} \quad \pi/2 \le \theta' \le \pi. \tag{4}$$

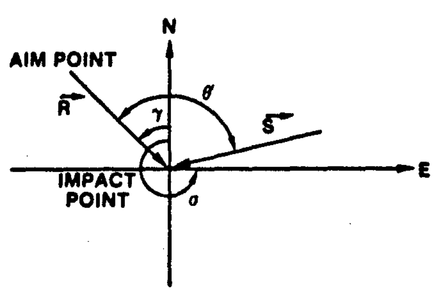

Figure 2. Initial angle parameters.

2.2 Early and Late Impacts

To define criteria for early and late impacts, the surface projection of the pre-entry trajectory is first assumed to be linear. Let L be the line in the surface plane through the aim point and normal to the surface trajectory. The impact is defined to be early if the surface trajectory does not cross L before impact. For example, the impact point of path 1 in figure 3 is early. The impact is defined to be late if the surface trajectory has crossed L. The impact point of path 2 in figure 3, for example, is late. If the impact point is on L, the missile impact is defined as late also.

If the surface projection of the pre-entry trajectory is not rectilinear, the tangent line to the curve at the point of impact replaces the linear surface trajectory above. Now, L is the normal through the aim point to the tangent at the impact point. Early and late impacts are then defined as for a linear surface trajectory. This situation is illustrated in figure 4 for the case when the impact is late.

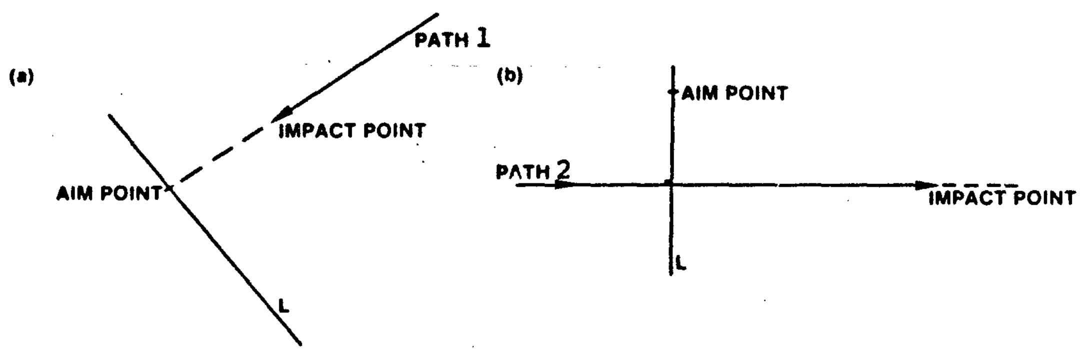

Figure 3. Impact in surface plane: (a) early versus (b) late.

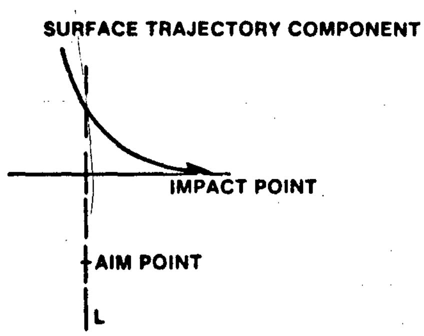

Figure 4. Curved trajectory impact type.

2.3 Criteria for Impact Form

$$\text{If } \theta' \leq \pi/2, \quad \text{the impact is late;} \tag{5}$$
$$\text{if } \theta' > \pi/2, \quad \text{the impact is early.} \tag{6}$$

The details of the derivation of equations (5) and (6) are given in appendix A. Briefly, a set of diagrams similar to figure 2 are analyzed for impact type and the corresponding form of θ'.

2.4 Derivation of Miss Distance

In the calculation of the miss distance, R, it is assumed that the impact angle, α, between the pre-entry trajectory and the surface, as well as the angles θ, γ, and σ are known. Then either h, the current height, or a, the distance along the trajectory to the impact point at any particular time before entry, is required to determine R. Also required is β, the angle between the trajectory and the line to the aim point (see fig. 5). These parameters are determined at some pre-impact time. But, since determination of R also requires information as to whether impact is early or late, the calculation of R must be done after impact. The two possibilities are depicted in figures 5 and 6. The segment f is constructed to be normal to the x axis. From these figures, the parameters a or h satisfy

$$a = h/\sin \alpha. \tag{7}$$

In both cases, the lengths b, f, and g are given by

$$b = a \cos \alpha \tag{8}$$

$$f = b \sin \theta \tag{9}$$

$$g = b \cos \theta. \tag{10}$$

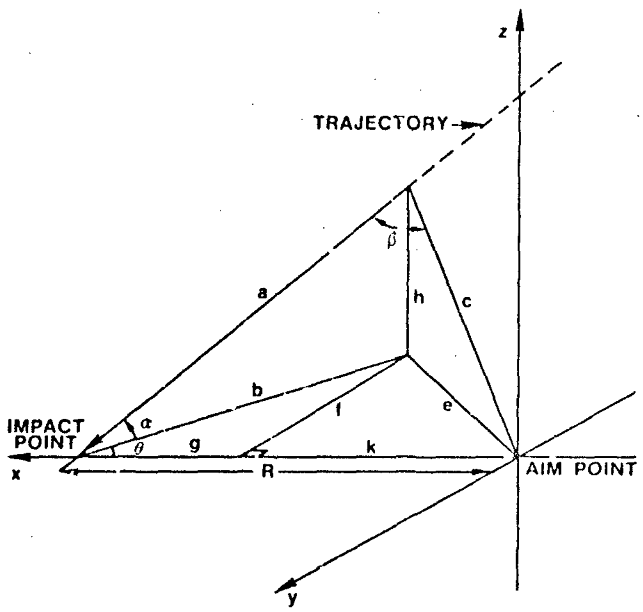

Figure 5. Late impact geometry, $\theta = \theta'$.

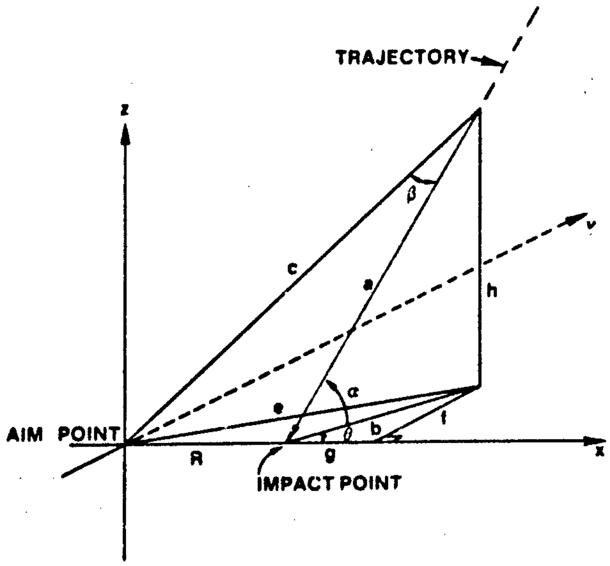

Figure 6. Early impact geometry, $\theta = \pi = \theta'$.

2.4.1 Derivation for Early Impact

Given

$$a^2 = b^2 + h^2, \tag{11}$$

$$c^2 = h^2 + e^2, \tag{12}$$

$$b^2 = g^2 + f^2, \tag{13}$$

$$e^2 = f^2 + (R + g)^2, \quad \text{and} \tag{14}$$

$$R^2 = a^2 + c^2 - 2ac\cos\beta. \tag{15}$$

From equations (12) and (14),

$$c^2 = h^2 + e^2 = h^2 + f^2 + g^2 + 2Rg + R^2. \tag{16}$$

Using equations (11) and (13) to simplify,

$$c^2 = a^2 + 2Rg + R^2 \tag{17}$$

Substituting equation (17) into equation (15) gives

$$R^2 = a^2 + (a^2 + 2Rg + R^2) - 2ac\cos\beta, \quad \text{and} \tag{18}$$

$$ac\cos\beta = (a^2 + Rg). \tag{19}$$

By equation (17),

$$(a^2 + 2Rg + R^2)^{1/2} = c = \frac{a^2 + Rg}{a\cos\beta}. \tag{20}$$

After both sides are squared, this simplifies to a quadratic in R,

$$(a^2\cos^2\beta - g^2)R^2 - (2ga^2\sin^2\beta)R - a^4\sin^2\beta = 0. \tag{21}$$

Solving this quadratic for R and simplifying,

$$R = \frac{ga^2 \sin^2 \beta}{(a^2 \cos^2 \beta - g^2)} \pm \frac{a^2 \sin \beta \cos \beta \sqrt{a^2 - g^2}}{(a^2 \cos^2 \beta - g^2)}. \tag{22}$$

2.4.2 Derivation for Late Impact

Given equations (11) through (13) as in the early case,

$$R = g + k \tag{23}$$

$$e^2 = f^2 + k^2, \quad \text{and} \tag{24}$$

$$R^2 = a^2 + c^2 - 2ac \cos \beta. \tag{25}$$

Then from equations (12), (24), and (23),

$$c^2 = h^2 + e^2 = h^2 + f^2 + k^2 = h^2 + f^2 + (R - g)^2. \tag{26}$$

Simplifying by equations (11) and (13),

$$c^2 = a^2 + R^2 - 2Rg. \tag{27}$$

Eliminating c^2 in equation (25),

$$R^2 = a^2 + (a^2 + R^2 - 2Rg) - 2ac \cos \beta, \quad \text{and} \tag{28}$$

$$ac \cos \beta = a^2 - Rg. \tag{29}$$

Squaring and using equation (27),

$$c^2 = a^2 - 2Rg + R^2 = \frac{(a^2 - Rg)^2}{a^2 \cos^2 \beta}. \tag{30}$$

Rewriting as a quadratic in R,

$$(a^2 \cos^2 \beta - g^2)R^2 + 2a^2 g \sin^2 \beta \, R - a^4 \sin^2 \beta = 0. \tag{31}$$

Solving for R as before and simplifying,

$$R = -\left[\frac{ga^2 \sin^2 \beta}{(a^2 \cos^2 \beta - g^2)} \pm \frac{a^2 \sin \beta \cos \beta}{(a^2 \cos^2 \beta - g^2)} \sqrt{a^2 - g^2}\right]. \tag{32}$$

Combining the early and late cases,

$$R = \pm \frac{a^2 \sin \beta (g \sin \beta \pm \cos \beta \sqrt{a^2 - g^2})}{(a^2 \cos^2 \beta - g^2)} \tag{33}$$

where the (+) value of the first (±) sign corresponds to an early impact and the (−) value to a late impact.

From figures 5 and 6,

$$g = b \cos \theta,$$

$$b = a \cos \alpha.$$

So in terms of α, θ, and β,

$$R = \pm aK[L \pm \cos \beta \sqrt{M}]. \tag{34}$$

where a is known or $a = h/\sin\alpha$ if h is known,

$$K = \sin\beta/(\cos^2\beta - \cos^2\alpha\cos^2\theta),$$

$$L = \cos\alpha\cos\theta\sin\beta, \quad \text{and}$$

$$M = 1 - \cos^2\alpha\cos^2\theta.$$

The initial data from the guidance sensors and the necessary parameters subsequently calculated are summarized as follows.

1. Necessary logic inputs

α	impact angle
σ,γ	map system angles
V	impact velocity
A	deceleration after entry
R	miss distance: (a) given (b) calculated from equation (34)

2. Further parameters to be calculated

θ', the smaller positive angle between $\hat{R}$ and $\hat{S}$

(a) $\theta' = |\sigma - \gamma|$, if $|\sigma - \gamma| \le \pi$

(b) $\theta' = 2\pi - |\sigma - \gamma|$, if $2\pi \ge |\sigma - \gamma| > \pi$

(c) $0 \le \theta' \le \pi$

θ, the acute angle defined by

(a) $\theta = \theta'$ if $\theta' \le \pi/2$

(b) $\theta = \pi - \theta'$ if $\pi/2 \le \theta' \le \pi$

3. POINT TARGET

Using the pre-entry data, the information necessary to define the fuzing requirements can be derived. A minimum burst depth, m, and the values for α, θ, γ, σ, R, d, and V are assumed to be known.

3.1 Case I: Linear Underground Path to Target

For a linear underground path to target, a linear path both above and below ground and a constant deceleration are assumed (see fig. 7). The parametric equations of the position of the missile as a function of time are

$$x(t) = x_0 + V_x t + \frac{A_x}{2} t^2, \tag{35}$$

$$y(t) = y_0 + V_y t + \frac{A_y}{2} t^2, \quad \text{and} \tag{36}$$

$$z(t) = z_0 + V_z t + \frac{A_z}{2} t^2, \tag{37}$$

where A_x, A_y, A_z are the components of the deceleration A, where t is assumed to be zero at time of impact, where $x_0 = R$, $y_0 = 0$, and $z_0 = d$, and where

$$V_x = V \cos \alpha \cos \theta, \tag{38a}$$

$$V_y = V \cos \alpha \sin \theta, \quad \text{and} \tag{38b}$$

$$V_z = V \sin \alpha, \tag{38c}$$

and similarly,

$$A_x = A \cos \alpha \cos \theta, \tag{39a}$$

$$A_y = A \cos \alpha \sin \theta, \quad \text{and} \tag{39b}$$

$$A_z = A \sin \alpha \quad (\text{where } A < 0). \tag{39c}$$

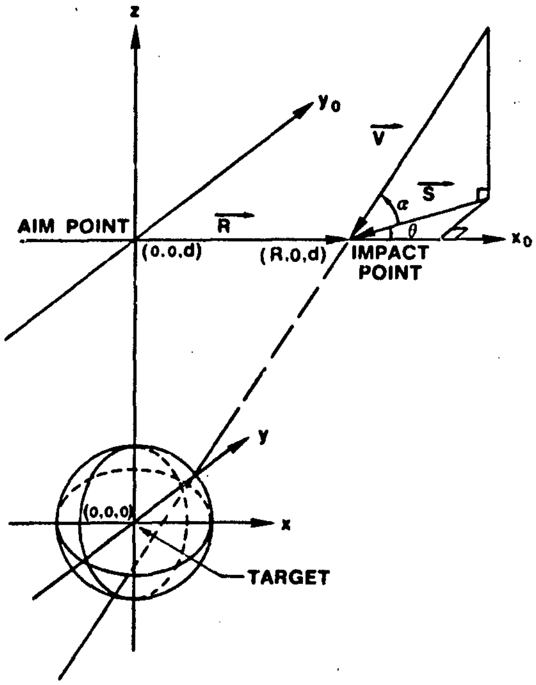

Figure 7. Linear underground trajectory to isovulnerable spherical volume about point target.

Next the specific parametrizations are defined considering the direction of change in each of the components of the missile position. Since $z(t)$ is always a decreasing function,

$$z(t) = d - V \sin \alpha \, t + \frac{A}{2} \sin \alpha t^2. \tag{40}$$

The function $y(t)$ is of the form:

$$y(t) = \pm V \cos \alpha \sin \theta t \pm \frac{A}{2} \cos \alpha \sin \theta t^2, \tag{41}$$

where the upper sign pertains to y increasing, and the lower to y decreasing. The function $x(t)$ is of the form

$$x(t) = R \mp \frac{A}{2} \cos\alpha \cos\theta t^2 \mp V \cos\alpha \cos\theta t, \tag{42}$$

where the upper sign pertains to early impact, and the lower to late impact. As the missile moves along its underground trajectory, the data processor must determine when the missile will be closest to the target. Further, the time to reach the minimum burst depth must be determined. For both quantities some form of distance function must be used. The distance from the target origin to a given point on the subsurface trajectory is given by $\rho(t) = \sqrt{x^2(t) + y^2(t) + z^2(t)}$. To minimize $\rho(t)$ it is sufficient to minimize $\rho^2(t)$, by setting the time derivative equal to zero. The time at which the missile is at the PCA is defined as τ. Computing $x^2(t)$, $y^2(t)$, and $z^2(t)$ from equations (40) through (42) gives the expression for $\rho^2(t)$ in equation (43) below. Consequently, the expression for $\rho^2(t)$ is not affected by the possible increase or decrease in either the x or y component. However, the early and late impact cases do influence the expression. The upper signs pertain to an early impact situation; the lower, to a late impact situation. This sign convention will be followed throughout the discussion unless otherwise noted.

$$\rho^2(t) = \frac{A^2}{4} t^4 + AVt^3 + (V^2 \mp RA \cos\theta \cos\alpha - Ad \sin\alpha)t^2 + (\mp 2VR \cos\theta \cos\alpha - 2Vd \sin\alpha)t + R^2 + d^2. \tag{43}$$

Then $\frac{d\rho^2}{dt} = 0$ implies

$$0 = A^2t^3 + 3AVt^2 + 2(V^2 \mp RA \cos\theta \cos\alpha - Ad \sin\alpha)t + (-2Vd \sin\alpha \mp 2VR \cos\alpha \cos\theta). \tag{44}$$

Dividing A^2, substituting $x - V/A$ for t, and solving for x in equation (44) gives

$$x = 0$$

or

$$x = \pm \frac{1}{A} \sqrt{V^2 + 2A(\pm R \cos\alpha \cos\theta + d \sin\alpha)}. \tag{45}$$

Then τ must be chosen from one of these values:

$$t_1 = -V/A + \frac{1}{A} \sqrt{V^2 + 2A(\pm R \cos\theta \cos\alpha + d \sin\alpha)}, \tag{46}$$

$$t_2 = -V/A, \quad \text{or} \tag{47}$$

$$t_3 = -V/A - \frac{1}{A} \sqrt{V^2 + 2A(\pm R \cos\theta \cos\alpha + d \sin\alpha)}. \tag{48}$$

For each of these three values, the function $\rho^2(t)$ has an extreme point. The value for t_1 represents the time to reach the first local minimum of $\rho^2(t)$. Next, t_2 corresponds to the time required for the missile to reach its maximum penetration and stop. The value of t_3 represents the improbable situation of the missile having negative velocity, retracing the path out of the earth, and passing through the local minimum again. To determine when to detonate the warhead so that all necessary conditions are satisfied, the time, τ, to reach the closest achievable point to the target must first be calculated. Then the minimum burst-depth conditions must be incorporated to finally arrive at the desired detonation time.

In determining τ, several possible situations arise. First, the PCA may be reached at or above the surface. In this case $\tau \leq 0$. Once the missile is beneath the surface, it may reach the PCA before reaching its maximum penetration. In this case $\tau = t_1$. On the other hand, the missile might stop at its maximum depth before achieving its PCA. In this case $\tau = t_2$. To determine which of these two realistic times, t_1 or t_2, is the time to reach the closest achievable point, two critical miss distances, R_1 and R_2, are necessary.

The parameters α, θ, d, A, and V determine a family of parallel trajectories. The miss distance determines a unique member of a family. A trajectory with miss distance R_1 will be at the PCA upon impact. For the PCA to be on the surface, $t_1 = 0$. That is,

$$-\frac{V}{A} = -\frac{1}{A}\sqrt{V^2 \pm 2A\cos\alpha\, R_1 + 2Ad\sin\alpha}\,. \tag{49}$$

Squaring and simplifying,

$$0 = \pm 2AR_1\cos\alpha\cos\theta + 2Ad\sin\alpha\,. \tag{50}$$

Thus,

$$R_1 = \mp\frac{d\sin\alpha}{\cos\alpha\cos\theta}\,. \tag{51}$$

On the other hand, a trajectory with miss distance R_2 will reach the PCA at the maximum penetration depth. Hence R_2 will depend upon the maximum depth as well as the initial parameters. For the PCA to be at the maximum depth, $t_1 = t_2$. Therefore, the radical in equation (46) must be zero.

$$V^2 \pm 2AR_2\cos\alpha\cos\theta + 2Ad\sin\alpha = 0 \tag{52}$$

Solving for R_2,

$$R_2 = \mp\left[\frac{V^2 + 2Ad\sin\alpha}{2A\cos\alpha\cos\theta}\right] \tag{53}$$

A particular maximum penetration depth, M, forces $R_2 = 0$. From figure 8,

$$\frac{M}{J} = \sin\alpha = \frac{J}{d} \tag{54}$$

where J is as shown in figure 8. So $M = d\sin\alpha^2$.

The constraint $R_2 = 0$ implies $\sin\alpha = -\dfrac{V^2}{2Ad}$ from equation (52). So

$$M = \frac{V^4}{4A^2d}\,. \tag{55}$$

However, referring to figure 9, for an early impact, any maximum penetration, μ, $\mu \leq M$, is equivalent to $R_2 \leq 0$ and $\mu > M$ is equivalent to $R_2 > 0$. Referring to figure 10, for a late impact, $\mu \leq M$ is equivalent to $R_2 > 0$, and $\mu \geq M$ is equivalent to $R_2 \leq 0$. Thus, M is not necessary for the analysis. Only the sign of R_2 is pertinent to the analysis.

Then, referring to figure 9, for an early impact, the processor calculates the value of R_2 using equation (53) with the upper signs. If $R_2 \leq 0$, then all early impacts will reach maximum penetration before the PCA. Thus, $\tau = t_2$. If the miss distance parameter R satisfies $0 < R < R_2$,

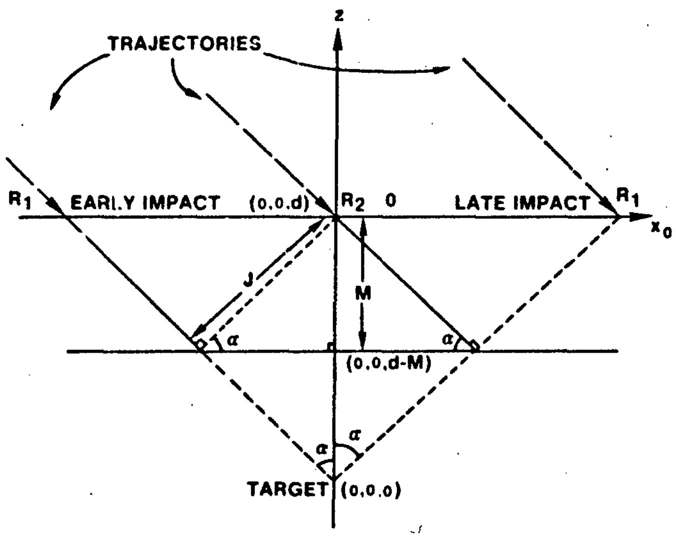

Figure 8. Maximum penetration depth at which $R_2 = 0$.

the missile will reach the PCA before stopping. Then $\tau = t_1$. If $R \geq R_2$, the PCA is below the maximum penetration depth. Hence, $\tau = t_2$.

Referring to figure 10 for a late impact, the data processor computes the values of R_1 and R_2 using equations (51) and (53) with the lower signs. Three cases arise when $R_2 > 0$. If $0 \leq R < R_2$, $\tau = t_2$; the missile reaches its maximum depth before the PCA. If $R_2 \leq R < R_1$, the missile reaches the PCA above the maximum depth, so $t_1 = \tau$. If $R \geq R_1$ the PCA is reached before

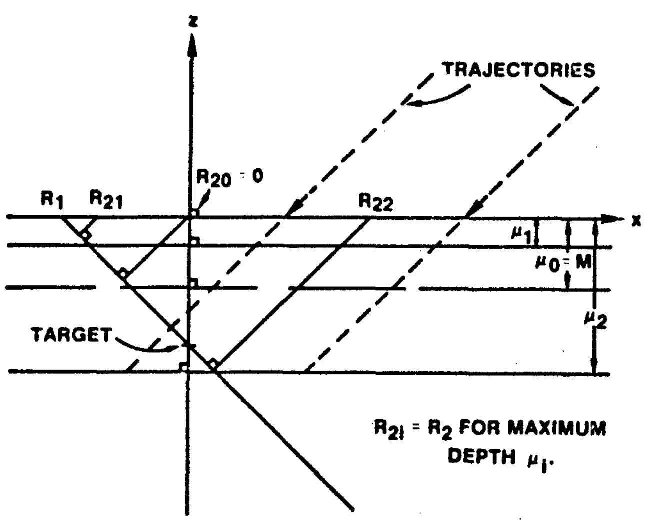

Figure 9. Critical miss distances for early impacts with various maximum penetration depths.

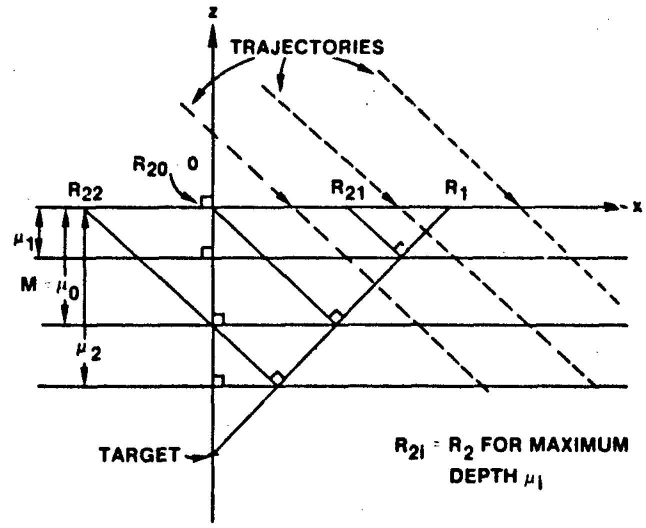

Figure 10. Critical miss distances for late impacts with various maximum penetration depths.

impact and $\tau \leq 0$. Two cases occur for $R_2 \leq 0$. If $R > R_1$, the PCA is again aboveground. But if $0 < R < R_1$, the PCA is reached below the surface but above the maximum depth. In this case $\tau = t_1$.

To illustrate this behavior, dependent on two critical miss distances, the HP9810 was programmed to plot $\rho^2(t)$. In figure 11, $\rho^2(t)$ is plotted for various miss distances (noted above the curves) using the following values: $d = 30.5$ m, $\alpha = \theta = \pi/3$, $A = -3.6 \times 10^4$ m/s^2 and $V = 610$ m/s. Assuming an early impact, the critical miss distance $R_2 = 8.95$ m.

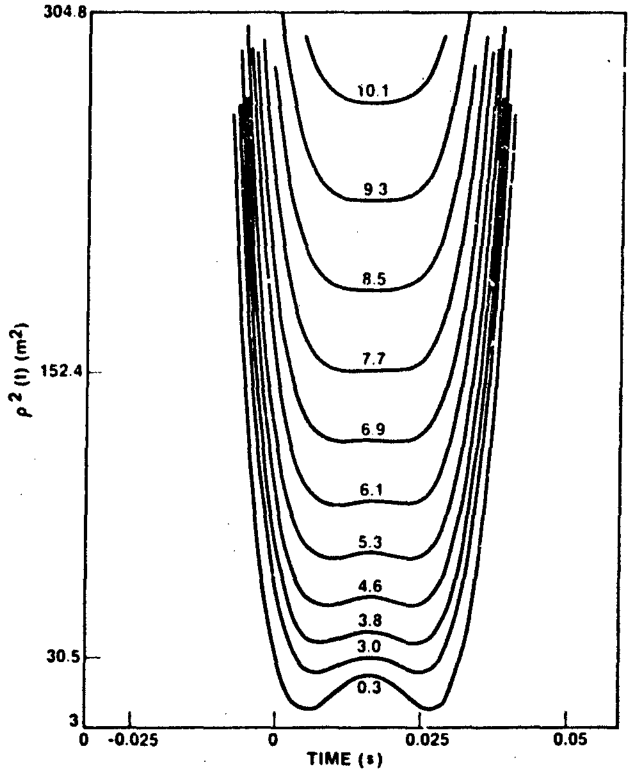

Figure 11. The effect of R on target range as function of time.

There is a minimum desired burst depth constraint to be considered also. The time, T, to reach this depth, m, is the solution to

$$z(T) = m = -\frac{A}{2} \sin \alpha \, T^2 - V \sin \alpha T + d. \tag{56}$$

Solving for T gives

$$T = \frac{V}{A} \pm \frac{\sqrt{V^2 \sin^2 \alpha + 2A(d - m) \sin \alpha}}{(-A \sin \alpha)}. \tag{57}$$

The time, T, must be the smaller positive value since the larger corresponds to a negative missile

velocity. Assuming that $d > m$, that is, that the target is below the minimum depth, then

$$(2A \sin \alpha)(d - m) < 0 \tag{58}$$

since $A < 0$. Therefore, $V \sin \alpha$ is greater than the radical. So

$$T = -\frac{V \sin \alpha + \sqrt{V^2 \sin^2 \alpha + 2A(d - m) \sin \alpha}}{A \sin \alpha}. \tag{59}$$

The minimum burst-depth criterion forces the logic (see fig. 12) to state that detonation occurs at τ when $\tau \geq T$. Otherwise detonation occurs at T.

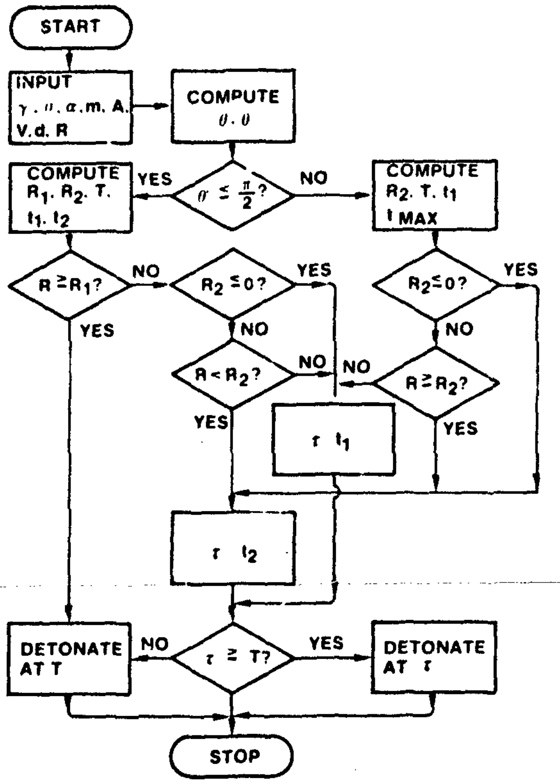

Figure 12. Logic diagram for linear underground path to point target.

3.2 Case II: Linear Underground Path to Isovulnerable Sphere

In this case, we consider a linear underground trajectory where a detonation is desired at the PCA to an isovulnerable sphere. The target is assumed to be inside an isovulnerable sphere of known radius r. The depth to the center of the sphere is now d. The values of A, V, R, α, θ, γ, σ, and d are known. The procedure is essentially the same as in Case I with the new objective being detonation as close as possible to the isovulnerable sphere. The distance function $\rho(t)$ and the position functions $x(t)$, $y(t)$, and $z(t)$ now are with respect to the center of the sphere. In addition to the logic of Case I, the fuze can detonate at any time t for which $\rho(t) \leq r$. The times T_1 and T_2 are the times when $\rho(t) = r$, that is, when the missile pierces the isovulnerable sphere. These values are the two real minimal positive roots of $\rho^2(t) - r^2 = 0$. But the solution of the fourth-degree equation in time is quite involved. The procedure is outlined in appendix B and can be programmed with impact parameters, if so desired. The result is a time interval $[T_1, T_2]$ in which to detonate. A second method is for the fuze data processor to continually compute $\rho(t)$ and detonate shortly after the time T_1, at which $\rho(T_1) = r$.

However, for both Cases I and II, a third method for finding the PCA is to continuously compute and compare successive values of $\rho(t)$. When the $\rho(t)$ values stop decreasing and start increasing, the missile is at a PCA. If the missile is below the minimum burst depth, m, detonation can occur at this time. But there are two problems: (1) how many values are enough to ascertain that $\rho(t)$ has begun to increase, ruling out random error fluctuations, and (2) how to proceed if $\rho(t)$ is initially increasing. The former problem requires experimental data before an answer can be made. The latter problem could occur if α is so small that the missile reaches the PCA before the data processor can compute T and τ. This difficulty could be avoided by providing an additional condition to use the third method if $\rho(t)$ is initially decreasing. If not, detonation should occur at T.

The following is a summary of the logic for a linear underground path to a point target.

1. Input initial parameters m, γ, σ, A, V, α, and R. If R will not be available, input a or h and calculate R.
2. Compute
   (a) θ', θ,
   (b) form of impact,
   (c) appropriate critical miss distances, and
   (d) T, t_1, t_2.
3. Detonate
   (a) at $t = \tau$ if $\tau > T$, or
   (b) at $t = T$ if $\tau \leq T$.

4. First alternative to PCA fuzing
   (a) Proceed as in steps 1 through 2(b).
   (b) Monitor $\rho(\cdot)$.
   (c) When $\rho(t)$ reaches a local minimum, detonate, if $z(t) \leq m$.
   (d) If at this minimum $z(t) > m$, delay detonation until $z(t) \leq m$.
5. Second alternative, for lethal sphere considerations
   (a) Monitor $\rho(t)$ until $\rho(t) \leq r$.
   (b) Detonate when $z(t) \leq m$.

3.3 Case III: Nonlinear Underground Path

In this case, the initially linear trajectory after entry becomes a curve because of missile accelerations. As in the linear underground trajectory cases, it is assumed that the values of α, γ, σ, θ, R, V, m, and d are known before entry. If the target is within an isovulnerable sphere, the depth, d, becomes the depth to the center of the sphere. By hypothesis, the missile has accelerometers to measure the components of the acceleration A with respect to the target oriented coordinate system. For this discussion it will also be assumed that the missile remains underground after impact. Figure 13 illustrates the geometry for this case. The parametric equations of position with respect to time are

$$x(t) = \int_0^t \int_0^u A_x(s)\, ds\, du \mp V_x t + x_0 , \tag{60}$$

$$y(t) = \int_0^t \int_0^u A_y(s)\, ds\, du \mp V_y t + y_0 , \text{ and} \tag{61}$$

$$z(t) = \int_0^t \int_0^u A_z(s)\, ds\, du + V_z t + z_0 , \tag{62}$$

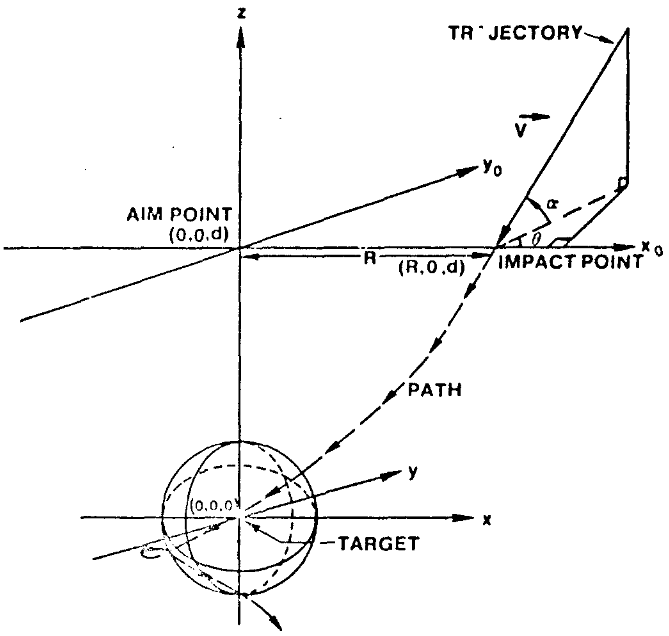

Figure 13. Nonlinear underground trajectory to point target at center of isovulnerable sphere.

where $(x_0, y_0, z_0) = (R, 0, d)$, and the velocity components are

$$V_x = V \cos\alpha \cos\theta,$$
$$V_y = V \sin\theta \cos\alpha, \quad \text{and}$$
$$V_z = V \sin\alpha.$$

Using $z(t)$ for depth, the missile will reach minimum burst depth, m, when

$$m = z(T) = \int_0^T \int_0^u A_z(s)\, ds\, du - (V \sin\alpha)\, T + d. \tag{63}$$

The data processor must compute this function continuously to determine when the missile is below the minimum depth.

Next, the distance between a point on the trajectory and the target is given by

$$\rho(t) = \sqrt{x^2(t) + y^2(t) + z^2(t)}. \tag{64}$$

After substitution this becomes

$$\rho(t) = \Bigg[\left(\int_0^t \int_0^u A_x(s)\, ds\, du \mp V \cos\alpha \cos\theta t + R\right)^2 + \left(\int_0^t \int_0^u A_y(s)\, ds\, du \mp V \cos\alpha \sin\theta t\right)^2 + \left(\int_0^t \int_0^u A_z(s)\, ds\, du - V \sin\alpha t + d\right)^2\Bigg]^{1/2}. \tag{65}$$

A first approach to the problem of minimizing $\rho(t)$ is to assume that the path has only one minimum. Then the data processor should calculate $\rho(t)$ values continually, as in the linear case, to determine when $\rho(t)$ is a minimum. If the missile is below the minimum depth at this point, then the warhead should be detonated. Otherwise, detonation should occur as soon as $z(t) = m$.

However, the trajectory may have several local minima. If the above procedure is used, the resultant minimum may not be the closest point achievable. One alternative would be to detonate at the first such minimum once below the minimum burst depth. Another alternative, which is useful if there is a isovulnerable sphere of known radius, r, would be to detonate when $\rho(t) \le r$. A third method, a combination of the above, would be to detonate when $\rho(t) \le r$ or when the path reaches a local minimum, whichever comes first after passing the minimum depth.

4. LINEAR TARGET WITH CYLINDRICAL ISOVULNERABLE VOLUME

As in the previous analysis, it is assumed that the values of α, θ, R, γ, σ, V, and m are known. If the target is a line, the analysis considers a cylindrical isovulnerable contour of length l and radius r, with hemispheres of radius r on the ends of the cylinder. Such a configuration is shown in figure 14. The total axial length is $l + 2r$. The depth to the top of this volume is d. The origin of the coordinate system is at the bottom of the volume so that the aim point on the surface has coordinates $(0, 0, D)$, where $D = 2r + l + d$. The objective is to detonate at the PCA to this volume. Assuming the missile is below the minimum depth, there are three possible points on the trajectory for detonation, depending on the depth of the missile. These possible detonation points are

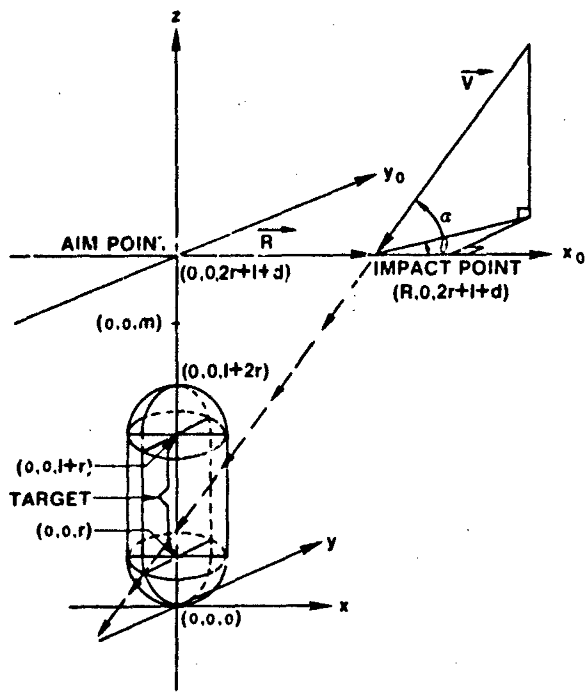

Figure 14. Linear underground trajectory to isovulnerable volume about linear target.

(1) the PCA to $(0, 0, l + r)$,

(2) the PCA to the z-axis if the z coordinate of the missile position satisfies $r < z < l + r$, and

(3) the PCA to $(0, 0, r)$ if the z coordinate of position is less than r.

For simplicity, it is assumed that detonation is to occur at the first such PCA reached below the minimum depth.

The parameterizations of the position of the missile are similar to those for the point target. First, assume the underground path is linear. Then,

$x(t)$ is given by equation (42),

$y(t)$ is given by equation (41), and (66)

$$z(t) = \frac{-A}{2} \sin \alpha t^2 - V \sin \alpha t + D.$$

Again, the time to reach the minimum burst depth, T, is obtained by solving $z(T) = m$. Thus

$$T = \frac{-V}{A} + \frac{\sqrt{V^2 \sin^2 \alpha + 2A(D - m) \sin \alpha}}{A \sin \alpha}. \tag{67}$$

To find the time to reach each of the three possible PCA's, three distance functions are required. $P(t)$ is defined to be the distance from any point (x, y, z) to $(0, 0, l + r)$. $M(t)$ is defined as the distance from any point to the portion of the z-axis between $(0, 0, l + r)$ and $(0, 0, r)$. $B(t)$ is defined as the distance from any point to the point $(0, 0, r)$.

The time to reach the point on the trajectory closest to $(0, 0, r + l)$, τ_1, is a solution to

$$\frac{d}{dt} P^2(t) = 0, \tag{68}$$

where

$$P^2(t) = x^2(t) + y^2(t) + [z(t) - (r + l)]^2. \tag{68}$$

After substituting for $x(t)$, $y(t)$, $z(t)$, and simplifying,

$$P^2(t) = \frac{A^2}{4}t^4 + AVt^3 + (V^2 \mp AR\cos\theta\cos\alpha - A(d + r)\sin\alpha)t^2 \tag{69}$$
$$+ 2V(\mp R\cos\alpha\cos\theta - (d + r)\sin\alpha)t + R^2 + (d + r)^2.$$

After setting the derivative equal to zero, equation (69) is in the same form as equation (44). So its solution is given by

$$\tau_1 = -\frac{V}{A} + \sqrt{V^2 + 2A\,[(d + r)\sin\alpha \pm R\cos\theta\cos\alpha]} \tag{70}$$

or

$$\tau_1 = -\frac{V}{A}.$$

The time to reach the point on the path closest to the z axis, τ_2, is a solution to

$$\frac{d}{dt} M^2(t) = 0,$$

where $M^2(t) = x^2(t) + y^2(t)$. That is,

$$M^2(t) = \frac{A^2}{4}\cos^2\alpha t^4 + AV(\cos^2\alpha)t^3 + (V^2\cos^2\alpha \mp RA\cos\alpha\cos\theta)t^2 \tag{71}$$
$$\mp 2VR(\cos\alpha\cos\theta)t + R^2.$$

After setting the derivative equal to zero and simplifying,

$$0 = t^3 + 3\frac{V}{A}t^2 + 2\left[\frac{V^2}{A^2} \mp \frac{R\cos\theta}{A\cos\alpha}\right]t \mp \frac{2VR\cos\theta}{A^2\cos\alpha}. \tag{72}$$

The same solution technique as for equation (44) gives the solutions:

$$\tau_2 = -\frac{V}{A}, \quad \text{or} \quad \tau_2 = -\frac{V}{A} \pm \frac{1}{A}\sqrt{V^2 \pm A\frac{R\cos\theta}{\cos\alpha}}. \tag{73}$$

The time to reach the point on the trajectory closest to the point $(0, 0, r)$, τ_3, is a solution to $\frac{d}{dt}B^2(t) = 0$, where

$$B^2(t) = x^2(t) + y^2(t) + (z(t) - r)^2.$$

In terms of initial parameters B^2 becomes

$$B^2(t) = \frac{A^2}{4}t^4 + AVt^3 + (V^2 \mp RA\cos\alpha\cos\theta - A(D - r)\sin\alpha)t^2 \tag{74}$$
$$+ 2V(-D\sin\alpha \mp R\cos\alpha\cos\theta)t + D^2 + R^2.$$

This expression for $B^2(t)$ is of the same form as equation (44) where d is $(D - r)$ here. The solutions are given by

$$\tau_3 = -\frac{V}{A} + \frac{1}{A}\sqrt{V^2 + 2A(D - r)\sin\alpha \pm R\cos\alpha\cos\theta}$$

or

$$\tau_3 = -\frac{V}{A}. \tag{75}$$

The objective is to detonate at the point of closest approach that the missile can achieve below the surface. The times τ_1, τ_2, and τ_3 are the times to the PCA's along an infinitely long trajectory. They will always occur in this order. However, the missile may reach these points before impact. Further, the missile may not reach any of these points at all if they are below the maximum attainable depth. Again $t = -V/A$ is the time at which the missile reaches the maximum depth. As in the point target case, there are certain critical miss distance values necessary to determine which of these times is optimal for detonation.

R_2 and R_4 are the miss distances for which the PCA's to $(0,0,l+r)$ and to $(0,0,r)$ respectively, occur at the maximum penetration depth. R_1 and R_3 are the miss distances for which the PCA's to $(0,0,l+r)$ and $(0,0,r)$, respectively, occur upon impact. These four critical miss distances are computed in the same way as their counterparts of the point target case. Thus,

$$R_4 = \mp\left[\frac{V^2 + 2A(D - r)\sin\alpha}{2A\cos\alpha\cos\theta}\right], \tag{76}$$

$$R_3 = \mp\left[\frac{(D - r)\sin\alpha}{\cos\theta\cos\alpha}\right], \tag{77}$$

$$R_2 = \mp\left[\frac{V^2 + 2A(d + r)\sin\alpha}{2A\cos\theta\cos\alpha}\right], \quad \text{and} \tag{78}$$

$$R_1 = \mp\left[\frac{(d + r)\sin\alpha}{\cos\theta\cos\alpha}\right]. \tag{79}$$

If the impact is early, R_1 and R_3 will both be negative. These critical miss distances also satisfy

$R_2 \geq R_4$ (otherwise $d > D$ which contradicts the hypothesis),

$R_2 \geq R_1$ (otherwise $V^2 < 0$), and

$R_4 > R_3$ (otherwise $V^2 < 0$).

Figures 15, 16, and 17 diagram the various relationships for an early impact. The procedure to determine the detonation time for an early impact is summarized as follows.

1. If $R_2 \leq 0$, then $R > R_2$ for any early impact and the missile stops before reaching a trajectory PCA. Therefore, detonation occurs at t_2.
2. If $R_2 > 0$, $R_4 > 0$, and R satisfies
   (a) $0 < R < R_4$, the missile reaches all trajectory PCA's in order. Detonation occurs at the first $\tau \geq T$. If there is no $\tau \geq T$, detonation occurs at T.
   (b) $R_4 < R < R_2$, the missile reaches the trajectory PCA to the top of the target first and then to the middle of the target. Detonation occurs at whichever $\tau \geq T$. Otherwise detonation occurs at t_2.

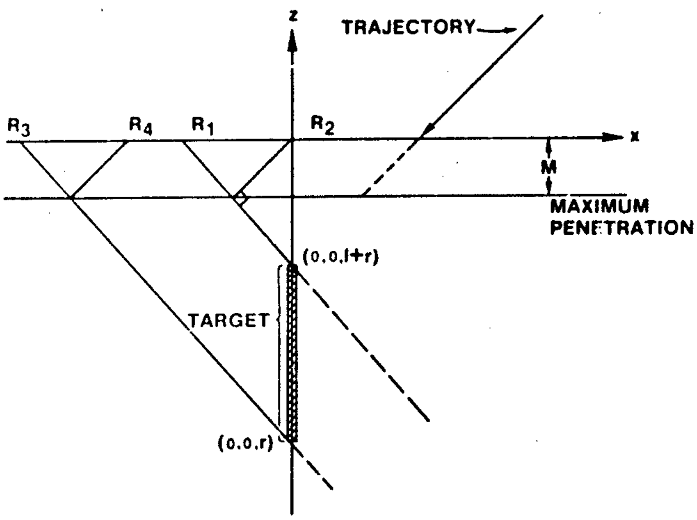

Figure 15. Early impact PCA relationships when $R_2 = 0$.

(c) $R_2 \leq R$, the missile passes through all the PCA's of the path on or before impact. Detonation occurs at T.

3. If $R_2 > 0$, $R_1 \leq 0$, and R satisfies
   (a) $0 < R < R_2$, the missile reaches the PCA with respect to the top first and to the z axis last. Detonation occurs at whichever $\tau \geq T$. Otherwise detonation occurs at t_2.
   (b) $R_2 \leq R$, the missile reaches its maximum penetration depth before any trajectory PCA. Detonation occurs at t_2.

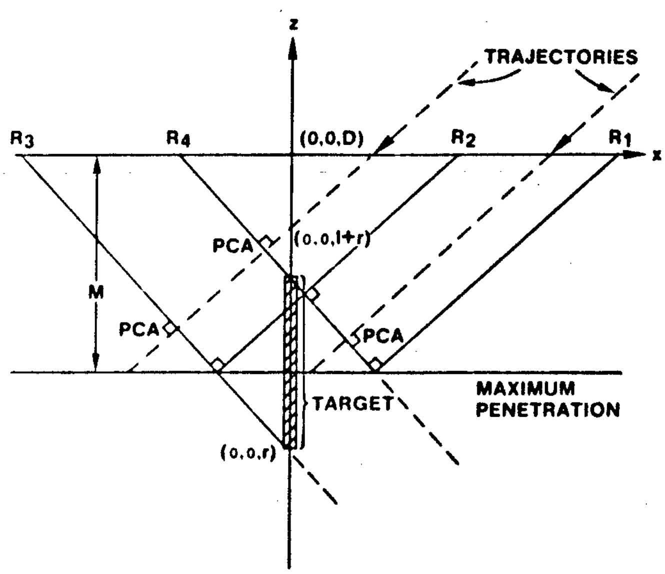

Figure 16. Early impact PCA relationships when $R_2 > 0$ and $R_4 > 0$.

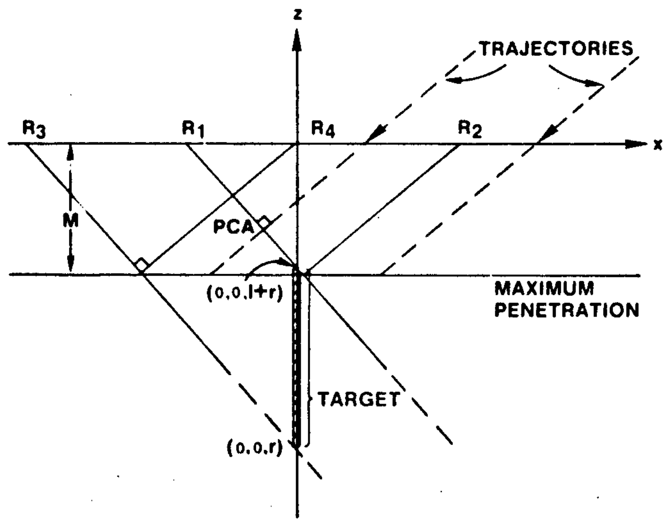

Figure 17. Early impact PCA relationships when $R_2 > 0$ and $R_4 = 0$.

On the other hand, if the missile is late, R_1 and R_3 are both positive. Further, these miss distances satisfy

$R_4 \geq R_2$ (otherwise $d > D$, contradicting the hypothesis)

$R_1 > R_2$ (otherwise $V^2 < 0$), and

$R_3 \geq R_4$ (otherwise $V^2 < 0$).

Figures 18, 19, and 20 diagram the various relationships for a late impact. The procedure to determine the detonation time for a late impact is summarized as follows.

1. If $R_4 \leq 0$ and R satisfies
   (a) $0 < R < R_1$, the missile reaches all the trajectory PCA's. Detonation occurs at whichever $\tau \geq T$ occurs first. Otherwise detonation occurs at T.
   (b) $R_1 \leq R < R_4$, the missile reaches the PCA to the top before impact. After impact it reaches the PCA to the z axis first and to the bottom last. Detonation occurs at τ_2 or τ_3, whichever is greater than T. Otherwise detonation occurs at T.
   (c) $R_4 < R$, all the PCA's of the path occur before impact. Detonation occurs at T.
2. If $R_4 > 0$, $R_2 \leq 0$, and R satisfies
   (a) $R < R_1$, the missile reaches first the PCA to the top, then to the z axis. Detonation occurs at τ_1, or τ_2, whichever is greater than T. If neither is greater than T, detonation occurs at t_2.
   (b) $R_1 \leq R < R_4$, the PCA to the top occurs before impact. After entry, only the PCA to the z axis occurs before the missile stops. Detonation occurs at τ_2 if $\tau_2 \geq T$. If not, detonation occurs at t_2.
   (c) $R_4 \leq R \leq R_3$, the missile after entry reaches the PCA to the z axis, followed by the PCA

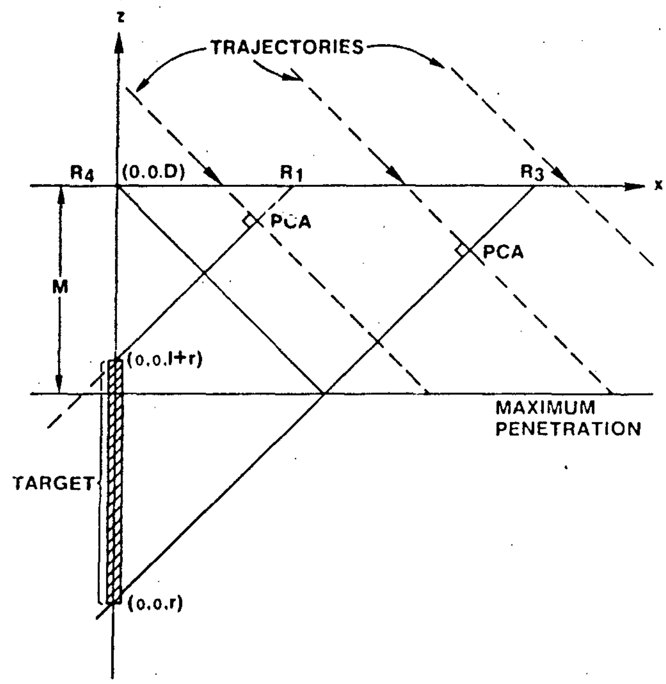

Figure 18. Late impact PCA relationships when $R_4 = 0$.

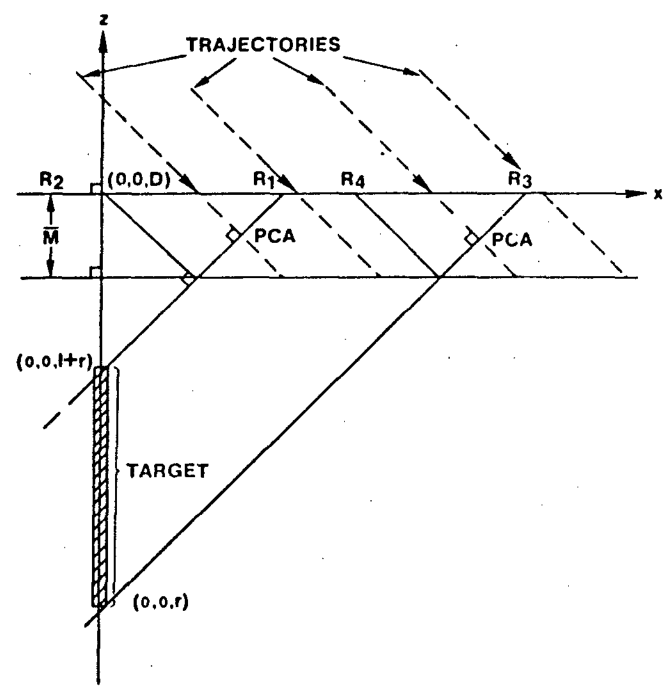

Figure 19. Late impact PCA relationships when $R_4 > 0$ and $R_2 = 0$.

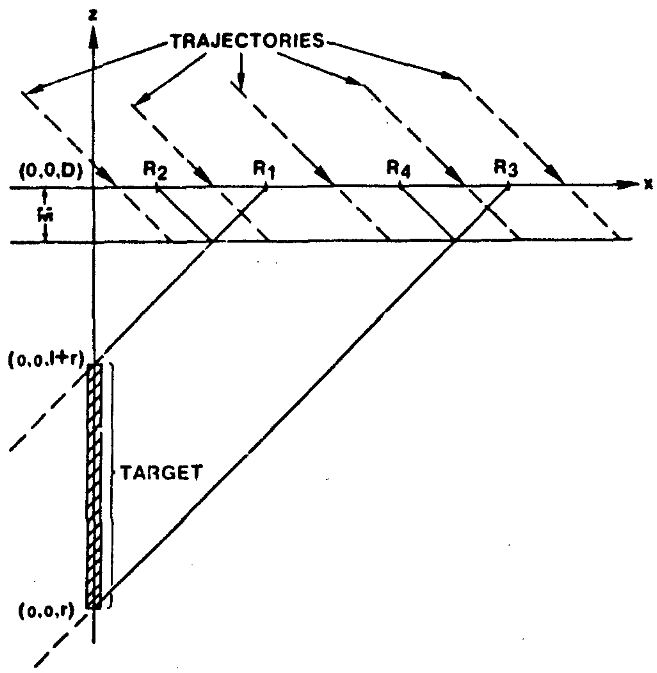

Figure 20. Late impact PCA relationships when $R_4 > 0$ and $R_2 > 0$.

to the bottom. Detonation occurs at τ_2 or τ_3, whichever is greater than T. Otherwise, detonation occurs at t_4.

(d) $R_3 < R$, all the PCA's of the path occur before impact. Hence detonation occurs at T.

3. If $R_4 > 0$, $R_2 > 0$, and R satisfies
   (a) $0 < R \leq R_2$, the missile reaches its maximum depth before any PCA. Detonation occurs then at t_2.
   (b) $R_2 \leq R \leq R_1$, the situation is the same as 2(a) for a late impact.

At this point, this case becomes identical to (2). Steps 3(c), (d), and (e) correspond to 2(b), (c), and (d). The complete logic flow chart for a linear path to a linear target is shown in figure 21.

With all other assumptions holding, now the underground path is taken to be some curve determined by the independent interaction of nonuniform component accelerations in the target coordinate system as shown in figure 22. Then the $x(t)$, $y(t)$, and $z(t)$ functions are found by integration of these accelerations. So $x(t)$, $y(t)$, and $z(t)$ are as defined in equations (60), (61), and (62), where

$$V_x = V \cos\alpha \cos\theta, \quad \text{and} \quad x_0 = R,$$
$$V_y = V \cos\alpha \sin\theta, \qquad y_0 = 0,$$
$$V_z = V \sin\alpha, \qquad z_0 = D = l + 2r + d.$$

To determine the depth of the missile, the data processor must evaluate $z(t)$. The minimum depth

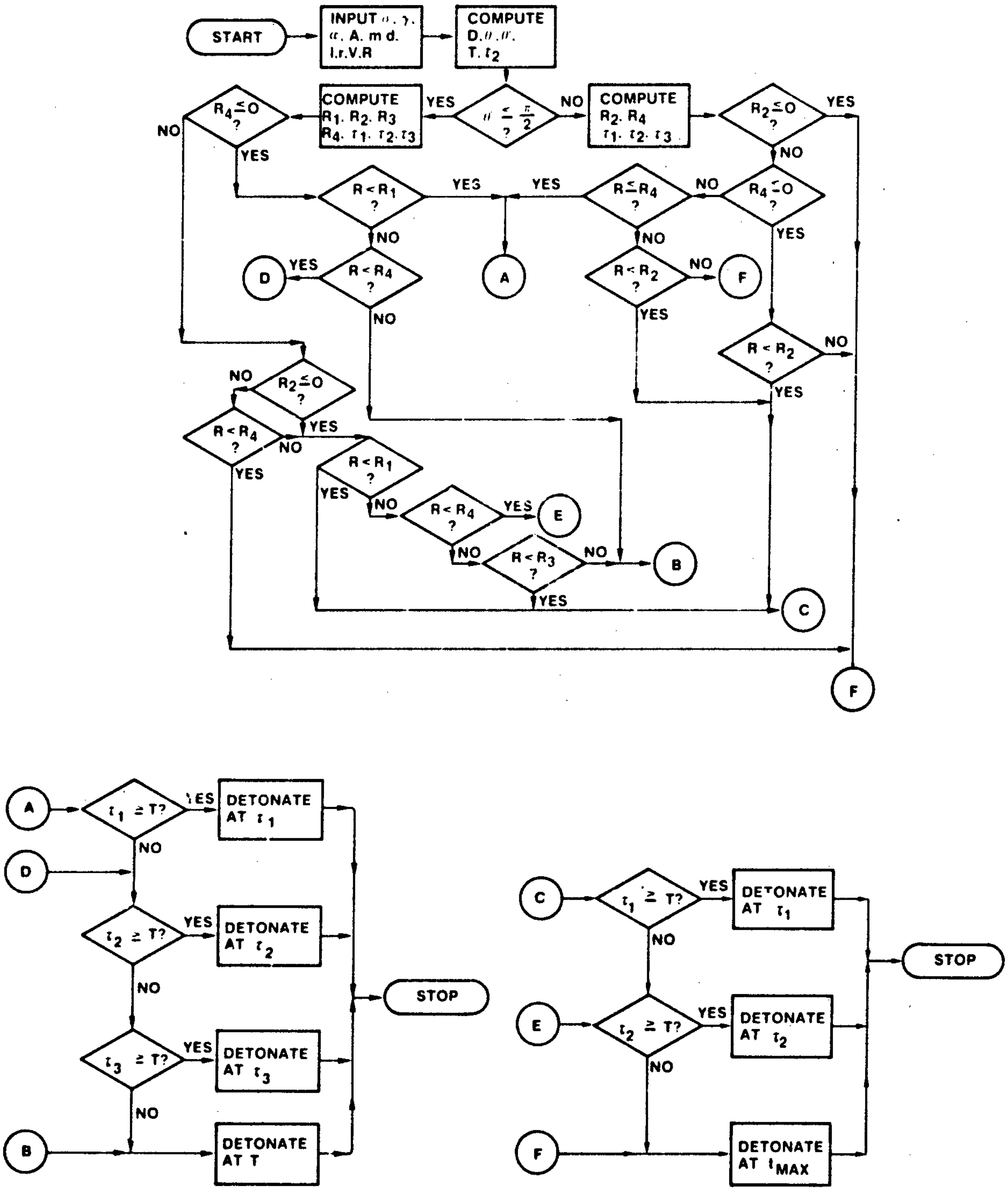

Figure 21. Logic diagram for a linear underground path to a linear target.

is attained when

$$z(T) - m = \int_0^T \int_0^u A_z(s)\, ds\, du - V(\sin\alpha)\, T + D - m = 0. \tag{80}$$

The distance functions $M^2(t)$, $P^2(t)$, and $B^2(t)$ are defined as before, but they are now expressed with the new forms of $x(t)$, $y(t)$, and $z(t)$. For example,

$$M^2(t) = \left[\int_0^t \int_0^u A_x(s)\, ds\, du \mp V(\cos\alpha\cos\theta)t + R\right]^2 + \left[\int_0^t \int_0^u A_y(s)\, ds\, du \mp V(\cos\alpha\sin\theta)t\right]^2. \tag{81}$$

There are two possible methods of determining the point for detonation but both entail monitoring the distance functions and $z(t)$, as outlined below.

1. Method A.
   (a) If $P^2(t)$, $M^2(t)$, and $B^2(t)$ are all increasing initially, detonate at T.
   (b) If $r + l < z(t) < m$ and $M^2(t)$ is increasing, detonate when $P^2(t) \le r^2$ or $P^2(t)$ changes from a decreasing to an increasing function, whichever occurs first below the minimum depth.
   (c) If $r \le z(t) \le l + r$, detonate when $M^2(t) \le r^2$ or when $M^2(t)$ changes from a decreasing to an increasing function, whichever occurs first below the minimum depth.

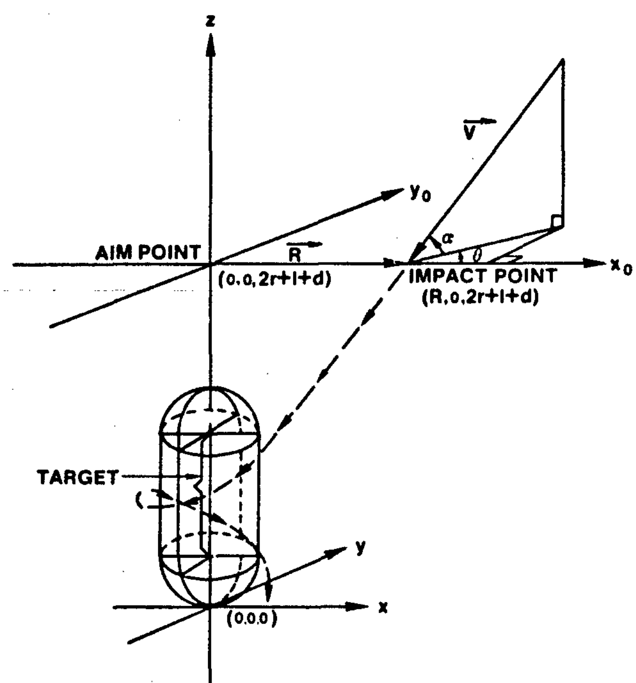

Figure 22. Nonlinear underground trajectory to isovulnerable volume about linear target.

(d) If $z(t) < r$, detonate when $B^2(t) \le r^2$ or when $B^2(t)$ changes from a decreasing to an increasing function, whichever occurs first below the minimum depth.
(e) Otherwise detonate when $z(t) = m$.

2. Method B.
(a) If $M^2(t)$, $B^2(t)$, and $P^2(t)$ are all initially increasing, detonate at T.
(b) If one of these three distance functions is initially decreasing, detonate when that function starts increasing, provided $z(t) \le m$.
(c) If more than one of these distance functions is decreasing initially, detonate once below the minimum depth at the first time for a function to change from decreasing to increasing.

5. CONCLUSIONS

The assumed objective of the fuzing algorithm is to detonate the warhead at the closest achievable point to the target when beneath the minimum burst depth. In the analysis it is assumed that the missile remains underground after entry. In most of the discussion all the necessary parameters are assumed to be known precisely. Terms that appear to be undefined as a parameter approaches zero are well defined if they are expanded as a series. The fuze algorithm results obtained for the assumption of only a known distribution of miss distance are briefly discussed in appendix B.

The general logic for the fuze in all cases is

(a) receive the impact data (α, γ, σ, V, A, $\hat{R}$, and d),
(b) determine which distance functions are appropriate,
(c) compute and choose the applicable time τ to reach a PCA,
(d) compute the time, T, to reach minimum depth,
(e) compare the values of T and τ, and then
(f) detonate at whichever time is the greater.

The logic for the linear path to a target can be analytically calculated for a deterministic answer. For the nonlinear case, the fuze must repetitively calculate various distance functions, comparing successive values with previous values to determine when the missile reaches a PCA or the minimum depth, or when the missile pierces the lethal volume.

For a complete model, much more must be incorporated into it. The fuzing logic should include allowances for (1) the prediction of a subsurface path from known soil characteristics, (2) a trajectory that oscillates above and below the ground, (3) nonsymmetric targets, and (4) inaccurate impact data. Further, the fuzing program must be flexible enough to be applicable to any form of target under the varied conditions of impact, soil, and data availability.

APPENDIX A
DIAGRAMS OF PARAMETER ANGLE RELATIONSHIPS

In the computations for the fuzing logic, the surface plane angle θ is required. Upon impact, though, only available are the map coordinate angles γ, measured counterclockwise from north to vector miss distance $\hat{R}$, and σ, measured counterclockwise from north to $\hat{S}$, the surface component of the trajectory. To formulate an expression for θ using γ and σ, as intermediate parameter θ' is required. The angle θ' is defined to be the smaller angle between $\hat{R}$ and $\hat{S}$. Further, θ' determines whether the impact is early or late.

Figure A-1(a) through (x) represents the surface map coordinate system at the point of impact. In each diagram a possible location of the aim point is plotted together with a vector representing a possible surface component of the trajectory. The map angles γ and σ are shown, as is the desired parameter θ'. Hence, figure A-1(a) through (x) represents all possible relations between σ, γ, and θ'. In each section of this illustration, θ' is obtained numerically in one of two ways.

$$\text{If } 0 \le |\sigma - \gamma| \le \pi, \quad \text{then} \quad \theta' = |\sigma - \gamma|.$$

$$\text{If } \pi \le |\sigma - \gamma| \le 2\pi, \quad \text{then} \quad \theta' = 2\pi - |\sigma - \gamma|.$$

In all cases, $0 \le \theta' \le \pi$.

Each of these figures, when analyzed as to the type of impact, early or late, yields criteria dependent on θ' for the type of impact. The results are shown in tables A-I and A-II.

TABLE A-I
Dependence of Impact Form on θ' when $\hat{S}$ and $\hat{R}$ are in Nonadjacent Quadrants

Figure A-1	Early/Late	$\theta' \left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\} \frac{\pi}{2}$	Figure A-1	Early/Late	$\theta' \left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\} \frac{\pi}{2}$
a	L	≤	j	L	≤
b	E	>	k	L	≤
c	L	≤	l	L	≤
d	E	>	m–t	See table A-II	
e	L	≤	u	E	>
f	E	>	v	E	>
g	L	≤	w	E	>
h	E	>	x	E	>
i	L	≤			

APPENDIX A

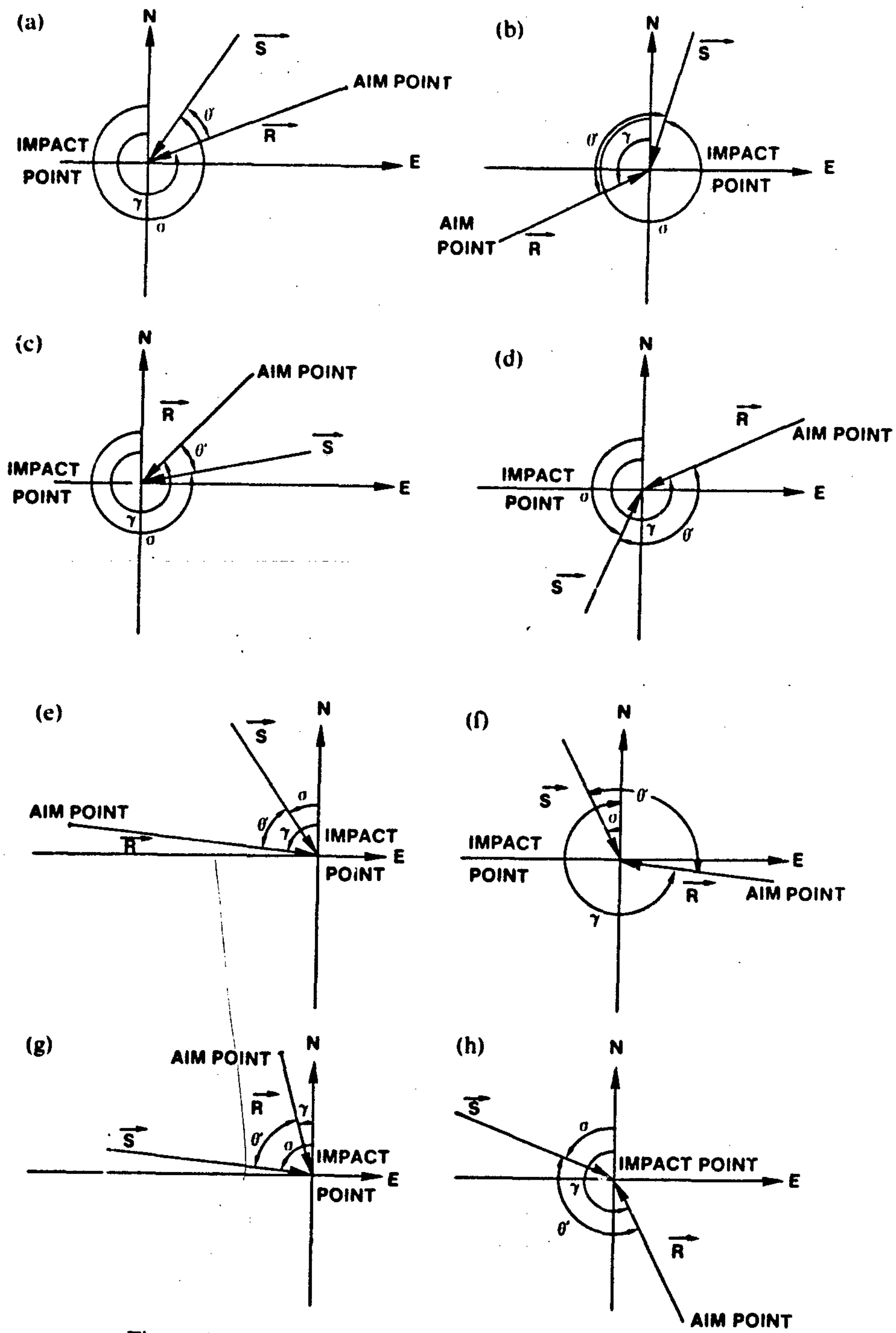

Figure A-1. Initial parameter angle relationships (cont'd).

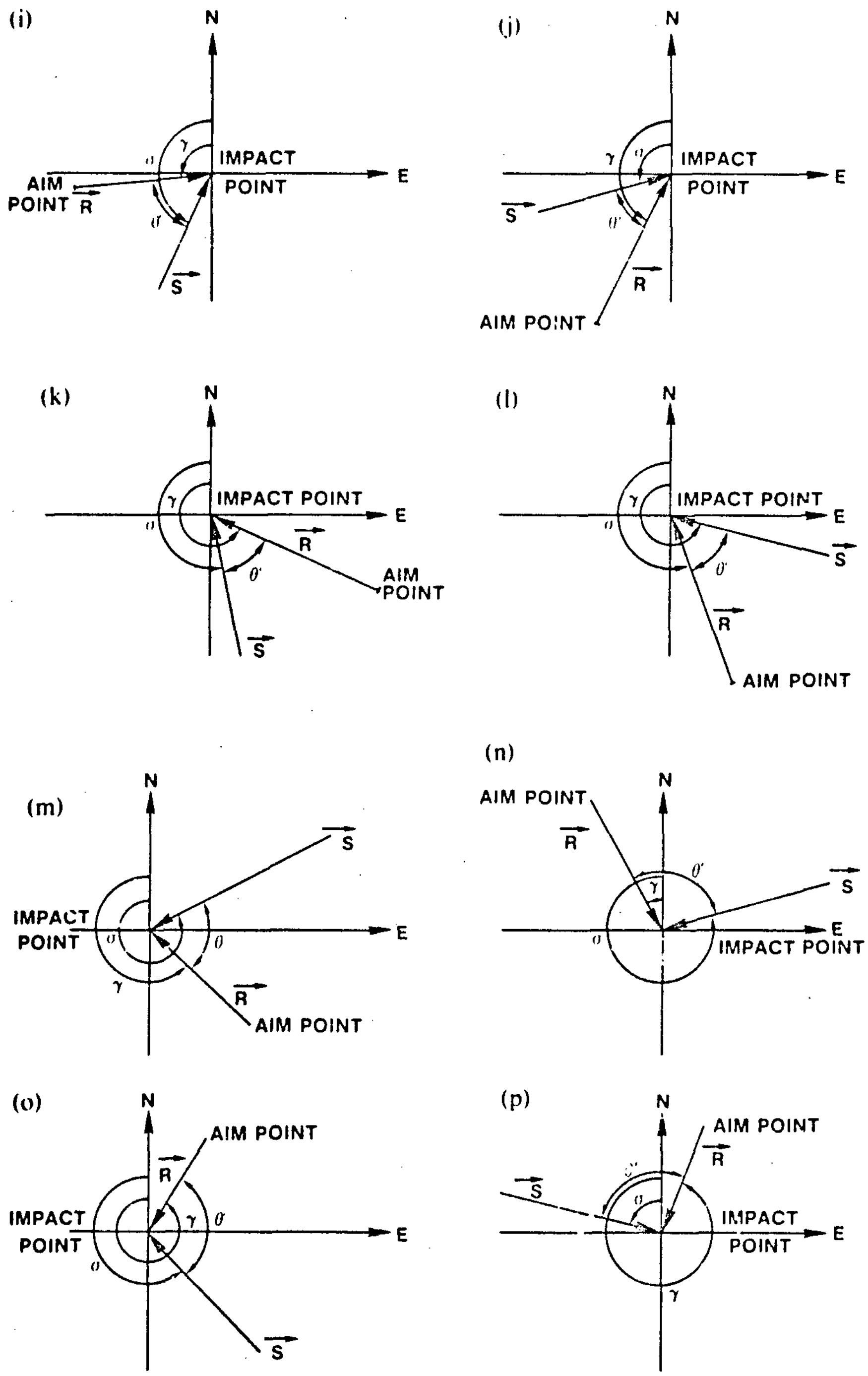

Figure A-1. Initial parameter angle relationships (cont'd).

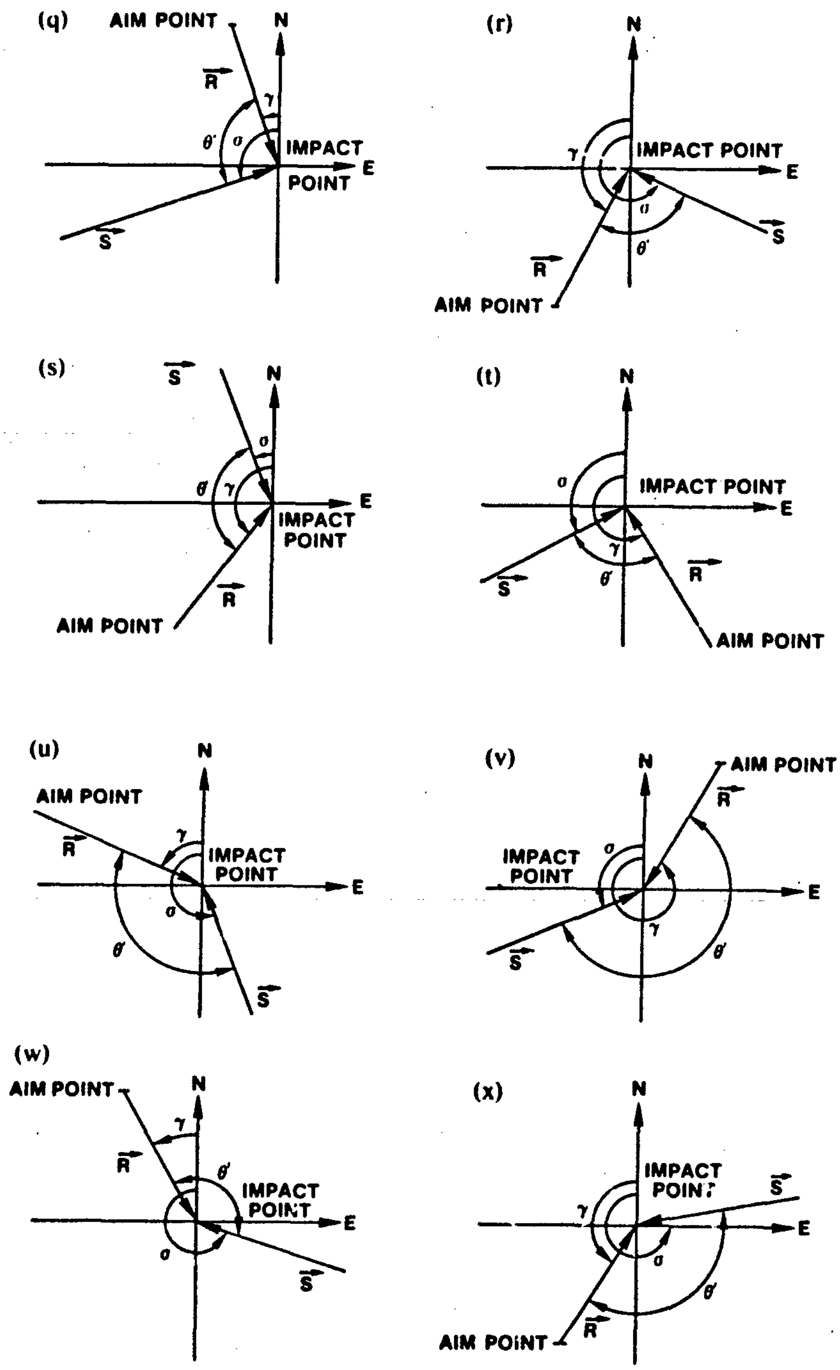

Figure A-1. Initial parameter angle relationships (cont'd).

TABLE A-II
Dependence of Impact Form on θ' when $\hat{S}$ and $\hat{R}$ are in Adjacent Quadrants

S_i	AP_i	Early/Late	$\theta' \left\{\begin{matrix}<\\=\\>\end{matrix}\right\} \frac{\pi}{2}$	S_i	AP_i	Early/Late	$\theta' \left\{\begin{matrix}<\\=\\>\end{matrix}\right\} \frac{\pi}{2}$
0	0	L	=	2	3	L	<
	1	L	<		4	L	<
	2	L	<				
	3	L	<	3	0	E	>
	4	L	<		1	E	>
					2	E	>
1	0	E	>		3	L	=
	1	L	=		4	L	<
	2	L	<				
	3	L	<	4	0	E	>
	4	L	<		1	E	>
					2	E	>
2	0	E	>		3	E	>
	1	E	>		4	L	=
	2	L	=				

Figure A-2, a composite generalization of figure A-1(m) through (t), shows the possible relationships between $\hat{S}$ and $\hat{R}$ when in adjacent quadrants. $\hat{R}_i$ is the vector miss distance from the ith aim point, AP_i, to the impact point. $\hat{S}_i$ is the surface component of the ith trajectory. Any relationship shown in figure A-1(m) through (t) is represented by a pair $(\hat{S}_i, \hat{R}_j)$ for some (i, j) pair where $i, j = 0, 1, 2, 3, 4$.

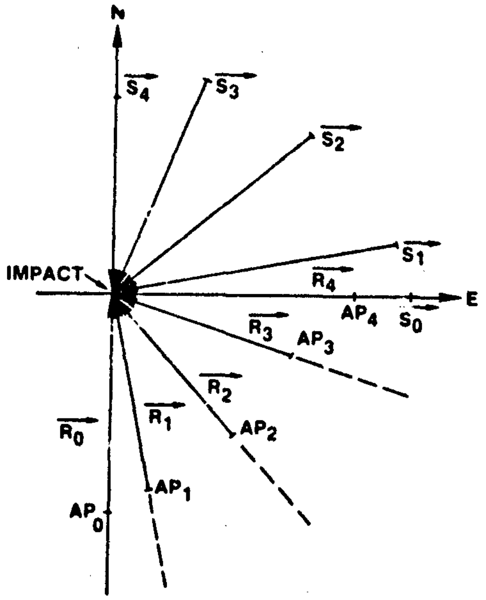

Figure A-2. Relations of $\vec{R}$ and $\vec{S}$ when in adjacent quadrants.

APPENDIX B

CALCULATION OF EXPECTATION FOR MISS DISTANCE

If the miss distance R is not known precisely, the values for R are assumed to be Rayleigh distributed with standard deviation σ and distribution function $F(R)$ given by

$$F(R) = \frac{R}{\sigma^2} e^{-R^2/2\sigma^2} \tag{B-1}$$

The analysis seeks the time t_1, to reach the point of closest approach. Since t_1 depends on R, if R is not known precisely, the expected value of R, $E(R)$, is used instead to calculate t_1. Using the variables V for the impact velocity, d for the depth to the target, A for the acceleration, α for the impact angle, and θ for the surface plane angle,

$$E(R) = \frac{-V}{A} + \frac{1}{A}\int_0^\infty [V^2 + 2A(\pm R\cos\theta\cos\alpha + d\sin\alpha]^{1/2} F(R)\, dR, \tag{B-2}$$

where (for $\pm$ and $\mp$ signs) the upper sign corresponds to an early impact and the lower to a late impact. Let $x = R$ and consider the integral

$$\int_0^\infty \sqrt{V^2 \pm 2xA\cos\alpha\cos\theta + 2Ad\sin\alpha}\left[\frac{x}{\sigma^2} e^{-x^2/2\sigma^2}\right] dx. \tag{B-3}$$

The data processor can estimate this integral by numerical methods. An alternative is to estimate (B-3) by

$$\int_0^\infty V\frac{x}{\sigma^2} e^{-x^2/2\sigma^2}\, dx. \tag{B-4}$$

Since the order of magnitude of V^2 is 10^9, while the other terms in the radicand contribute 10^2, the radical is approximated by V. But

$$\int_0^\infty \frac{Vx}{\sigma^2} e^{-x^2/2\sigma^2}\, dx = V. \tag{B-5}$$

So $E(R)$ is approximately $\frac{-V}{A} + \frac{V}{A} = 0$. The other times computed for the various cases all have this form. Therefore the expectation of each time will be zero, suggesting that this is too simple an approximation to equation (B-3). Numerical approximation, say by Simpson's Rule, appears to be a better method.

APPENDIX C

SOLUTION OF A FOURTH-DEGREE EQUATION

For a linear underground path to a lethal sphere of radius r, the object is to ascertain when the missile is within the lethal sphere. Hence the times, t, for which $\rho^2(t) \leq r^2$ are necessary where $\rho(t)$ is the distance to the center of the sphere as a function of time. To find such values of t, the equation $\rho^2(t) - r^2 = 0$ must be solved for t. Using the variables V for impact velocity, d for depth to the target, A for acceleration, α for impact angle, and θ for the surface plane angle, $\rho^2(t) - r^2 = 0$ becomes as follows.

$$0 = t^4 + \left[\frac{4V}{A}\right]t^3 + 4\left[\frac{V^2}{A^2} \mp \frac{R\cos\theta\cos\alpha \mp d\sin\alpha}{A}\right]t^2$$

$$\mp 8\left[\frac{V}{A^2}(R\cos\theta\cos\alpha + d\sin\alpha)\right]t + \frac{4}{A^2}(R^2 + d^2 - r^2), \tag{C-1}$$

where (for $\pm$ and $\mp$ signs) the upper sign corresponds to an early impact, and the lower to a late impact. This equation then is of the form

$$0 = t^4 + at^3 + bt^2 + ct + f \tag{C-2}$$

where a, b, c, and f are arbitrary constants. To solve this equation, first the following cubic equation is solved

$$y^3 - by^2 + (ac - 4f)y - a^2f + 4bf - c^2 = 0 \quad \text{where} \quad a, b, c, f \text{ are as in equation (C-2)} \tag{C-3}$$

To solve (C-4), let

$$\begin{aligned} p &= -b \\ q &= ac - 4f \\ r &= -a^2f + 4bf - c^2. \end{aligned} \tag{C-4}$$

Define

$$\begin{aligned} C &= \frac{1}{3}(3q - p^2) \\ B &= \frac{1}{27}(2p^3 - 9pq + 27r) \end{aligned} \tag{C-5}$$

Then a solution to (C-4) is $y = K + L - p/3$ where

$$K = \sqrt[3]{-\frac{B}{2} + \sqrt{\frac{B^2}{4} + \frac{C^3}{27}}} \quad \text{and} \quad L = \sqrt[3]{-\frac{B}{2} - \sqrt{\frac{B^2}{4} + \frac{C^3}{27}}} \tag{C-6}$$

Using this solution for y, determine R by evaluating

$$R = \sqrt{\frac{a^2}{4} - b + y} \tag{C-7}$$

If $R \neq 0$, then

$$F = \sqrt{\frac{3a^2}{4} - R^2 - 2b + \left(\frac{4ab - 8c - a^3}{4R}\right)} \tag{C-8a}$$

and

$$E = \sqrt{\frac{3a^2}{4} - R^2 - 2b - \left(\frac{4ab - 8c - a^3}{4R}\right)} \tag{C-8b}$$

...DING PAGE BLANK-NOT FILMED

APPENDIX C

If $R = 0$, then

$$F = \sqrt{\frac{3a^2}{4} - 2b + 2\sqrt{y^2 - 4f}} \qquad \text{(C-9a)}$$

and

$$E = \sqrt{\frac{3a^2}{4} - 2b - 2\sqrt{y^2 - 4f}} \qquad \text{(C-9b)}$$

Then the solutions of $\rho^2(t) - r^2 = 0$ are given by

$$\begin{aligned} t &= \frac{-a}{4} + \frac{R}{2} \pm \frac{F}{2} \\ t &= \frac{-a}{4} - \frac{R}{2} \pm \frac{E}{2} \end{aligned} \qquad \text{(C-10)}$$

The two real minimal positive values resulting, call them T_1, T_2, represent the times at which the missile trajectory enters and exits the isovulnerable volume. Thus, there is a time interval $[T_1, T_2]$ in which to detonate effectively.

DISTRIBUTION

DEFENSE DOCUMENTATION CENTER
CAMERON STATION, BUILDING 5
ALEXANDRIA, VA 22314
ATTN DDC-TCA (12 COPIES)

COMMANDER
USA RSCH & STD GP (EUR)
BOX 65
FPO NEW YORK 09510
ATTN LTC JAMES M. KENNEDY, JR.
CHIEF, PHYSICS & MATH BRANCH

COMMANDER
US ARMY MATERIEL DEVELOPMENT
& READINESS COMMAND
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333
ATTN DRXAM-TL, HQ TECH LIBRARY
ATTN DRCPA-S, SYS ANAL DIV
ATTN DRCPM, PROJ OFC, MIRADCOM

COMMANDER
US ARMY ARMAMENT MATERIEL
READINESS COMMAND
ROCK ISLAND ARSENAL
ROCK ISLAND, IL 61201
ATTN DRSAR-ASF, FUZE &
MUNITIONS SPT DIV

COMMANDER
USA MISSILE & MUNITIONS
CENTER & SCHOOL
REDSTONE ARSENAL, AL 35809
ATTN ATSK-CTD-F

DIRECTOR
DEFENSE ADVANCED RESEARCH
PROJECTS AGENCY
ARCHITECT BLDG
1400 WILSON BLVD
ARLINGTON, VA 22209
ATTN DIR, TACTICAL TECHNOLOGY OFFICE
ATTN DIR, TECHNOLOGY ASSESSMENTS OFFICE

DIRECTOR
DEFENSE NUCLEAR AGENCY
WASHINGTON, DC 20305
ATTN TACTICAL NUCLEAR DIV (RATN)
ATTN WEAPONS SYSTEMS DIV (VLWS)

DIRECTOR OF DEFENSE RESEARCH
& ENGINEERING
WASHINGTON, DC 20301
ATTN DEP DIR (TACTICAL WARFARE
PROGRAMS)
ATTN ASST DIR (LAND WARFARE)

OFFICE, CHIEF OF RESEARCH, DEVELOPMENT
& ACQUISITION
DEPARTMENT OF THE ARMY
WASHINGTON, DC 20310
ATTN DAMA-CSM-N, NUCLEAR TEAM
ATTN DAMA-WSM-S, SURFACE-TO-SURFACE TEAM

COMMANDER
US ARMY MATERIALS SYSTEMS
ANALYSIS ACTIVITY
ABERDEEN PROVING GROUND, MD 21005
ATTN DRXSY-CT, TAC OPS ANALYSIS

COMMANDER
US ARMY MISSILE RESEARCH
& DEVELOPMENT COMMAND
REDSTONE ARSENAL, AL 35809
ATTN DRCPM-PE, OFC PROJ MANAGER,
PERSHING II

COMMANDER
US ARMY ARMAMENT RESEARCH
& DEVELOPMENT COMMAND
DOVER, NJ 07801
ATTN DRDAR-FU, PRODUCT MANAGER, FUZES

HARRY DIAMOND LABORATORIES
ATTN RAMSDEN, JOHN J., COL, COMMANDER/
FLYER, I.N./LANDIS, P.E./
SOMMER, H./OSWALD, R. B.
ATTN CARTER, W.W., DR., TECHNICAL
DIRECTOR/MARCUS, S.M.
ATTN WISEMAN, ROBERT S., DR., DRDEL-CT
ATTN KIMMEL, S., PAO
ATTN CHIEF, 0021
ATTN CHIEF, 0022 (5 COPIES)
ATTN CHIEF, LAB 100
ATTN CHIEF, LAB 200
ATTN CHIEF, LAB 300
ATTN CHIEF, LAB 400
ATTN CHIEF, LAB 500
ATTN CHIEF, LAB 600
ATTN CHIEF, DIV 700
ATTN CHIEF, DIV 800
ATTN CHIEF, LAB 900
ATTN CHIEF, LAB 1000
ATTN RECORD COPY, BR 041
ATTN HDL LIBRARY (5 COPIES)
ATTN CHAIRMAN, EDITORIAL COMMITTEE
ATTN CHIEF, 047
ATTN TECH REPORTS, 013
ATTN PATENT LAW BRANCH, 071
ATTN GIDEP OFFICE, 741
ATTN LANHAM, C., 0021